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oxidation rate of annealed samples can be ascribed to the removal of internal stress caused by the rolling mill. There is no indication that protective oxidation was reduced by annealing. At higher temperatures the effect of annealing on emittance may be less significant due to thicker oxide layers in the absence of protective oxidation.

REFERENCES

- H. E. Bennet, Influence of surface roughness and surface damage, oxide films on emittance, NASA SPE-55, 145– 151 (1964).
- T. M. Donovan, E. J. Ashley and H. E. Bennet, Effect of surface damage on the reflectance of germanium in the 2650-1000 Å region, J. Soc. Am. Opt. 53, 1403-1409 (1963).
- P. J. Zanzucchi and M. T. Duffy, Surface damage and optical reflectance of single-crystal silicon, Appl. Optics 17, 3477-3481 (1978).
- R. R. Brannon, Jr. and R. J. Golstein, Emittance of oxide layers on a metal substrate, Trans. Am. Soc. Mech. Engrs, Series C, J. Heat Transfer 92, 257-263 (1970).
- H. Y. Wong and S. R. Aggarwal, Total hemispherical emittance of stainless steel AISI 321, J. Iron Steel Inst. 8, 635-637 (1971).

- A. G. Eubanks, D. G. Moore and W. A. Pennington, Effect of surface roughness on the oxidation of iron, J. Electrochem. Soc. 109, 382-389 (1962).
- C. Isetti and E. Nannei, Influence of surface treatments on the total normal emittance of AISI 316 stainless steel, Wärme- und Stoffübertragung 14, 211-215 (1980).
- G. Ruffino and C. Pisoni, An apparatus for emissivity measurements, High Temperatures—High Pressures 8, 419-424 (1976).
- C. Isetti, E. Nannei and C. Pisoni, The effect of oxidation and roughness on the total normal emittance of stainless steel, High Temperatures—High Pressures 11, 393-397 (1979).
- C. Isetti and E. Nannei, Total normal emittance of stainless steel at high temperature: influence of oxide layer growth, High Temperatures—High Pressures 12, 307-310 (1980)
- 11. O. Kubaschewski and V. S. Hopkins, Oxidation of Metal and Alloys. Butterworths, London (1962).
- D. J. McAdam and G. W. Geil, Rate of oxidation of steels as determined from interference colors of oxide films, J. Res. Natn Bur Stand. 23, 63-123 (1939).
- A. Brasunas, G. T. Gow and O. E. Harder, Resistance of iron-nickel-chromium alloys to corrosion in air at 1600 to 2000°F, Proc. Am. Soc. Test Mater. 46, 870-893 (1946).

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Corrigenda et addenda to two-component Bénard convection in cylinders

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In a RECENT paper [1] Crespo and Velarde present a linear stability analysis of Bénard convection in a nonreactive binary (gas or liquid) mixture in a vertical cylinder heated from below. Both asymmetric and axisymmetric convective patterns are considered although the main object of the paper refers to the former and its relevance to describe recent experimental data by Abernathey and Rosenberger [2]. In this Note we correct some misprints that slipped through while correcting the galley proofs and some numbers that unfortunately occurred in the final formula for axisymmetric convection thus leading to numerical errors in a Table of results. Notation and equations refer to the paper by Crespo and Velarde [1].

In equation (1.2) the term $SRa\Omega$ should be $SRa\Omega$. In equation (1.3) the term $D_F\nabla^2\Omega$ should have a minus sign. In equation (1.4) the bracket $(1+D_F)$ should have a plus sign. In Fig. 3 it should read $D_F = -S^2/10$ (with a minus sign). In equation (4.2) the expression $\partial [(1/r)\partial(ru)]/\partial r$ should be $\partial [(1/r)\partial(ru)/\partial r]/\partial r$. In equation (4.5) the factor S in front of the bracket in the fourth term should instead be r_D . In Fig. 5 the ordinate is Ra^c , as in Fig. 4.

In equation (4.11) some extra numbers appeared that unfortunately led to errors in the results reported in Table 1. The correct expression is

$$Ra^{c} = \{(200f^{2}/7\alpha^{2})[(1-\alpha^{2}/2)/f^{2}-21]$$

$$\times [5D_{F}\alpha^{4}/f^{4}-6(\alpha^{2}/f^{2}+10)\alpha^{2}/f^{2}]$$

$$+6(\alpha^{2}/f^{2}+10)^{2}\alpha^{2}/f^{2}$$

$$-5D_{F}(\alpha^{2}/f^{2}+10)\alpha^{4}/f^{4}\}$$

$$\times [6\alpha^{2}/f^{2}+5S(\alpha^{2}/f^{2}+10)/r_{D}$$

$$-5S\alpha^{2}/f^{2}-5D_{F}\alpha^{2}/r_{D}f^{2}]^{-1}.$$
(4.11)

Table 1. Number of axisymmetric rolls as a function of aspect ratio f = R/L. Large values of f approximate the infinitely extended horizontal layer. Note that for given aspect ratio the Soret effect induces a change of wavenumber. Results correspond to case $r_D = 0.01$

S f	0		0.01		0.1	
	Rac	Rolls	Rac	Rolls	Rac	Rolls
1	2975	1	1.418	1	248	1
2	1933	1	662	1	96	1
4	1903	3	638	2	76	1
7	1894	6	639	4	73	1
10	1889	9	639	6	73	1
100	1884	102	639	63	72	1

Note that, as in ref. [1], equation (4.11) accounts for both Soret and Dufour effects. The latter effect is disregarded in the following.

Corrections to equation (4.11) lead to corrections in Table 1. The correct and enlarged Table 1 is here given. Inspection of Table 1 shows that with a single trial function and a rather rough Galerkin calculation we obtain the one-roll structure in axisymmetric convection at S=0.1. The case f=100 is definitely a perfect approximation to the infinitely extended horizontal layer heated from below.

Although all possible applications are implicit in the equation (4.11) a straightforward analysis can help clarifying issues in some asymptotic limits. For instance, if we restrict consideration to the case of arbitrarily large aspect ratio the following results hold:

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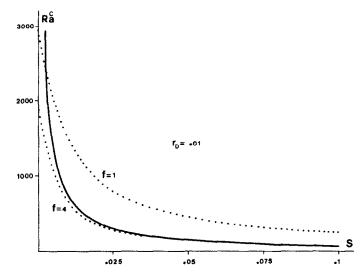


Fig. 1. Neutral stability curves in the Ra, S plane at $r_D = 0.01$, for different values of the aspect ratio, f = R/L. Note that for $S \gtrsim 0.029$ one cannot distinguish between the curves for $f \geqslant 4$ (dotted lines) and the approximation of Velarde and Schechter [3], $Ra^cS = 7.20$ (solid line). The dotted lines always cut the vertical axis.

(1) For S = 0 the result reported in [1] is not affected. The critical Rayleigh number is 1884 for a critical wavenumber of 3.22 which compares rather well with the more accurate results of 1707 and 3.14, respectively, already known in the literature.

(2) At vanishing r_D (κ arbitrarily larger than D) the combination Ra^cS/r_D approaches 720 for a vanishingly small wavenumber in accordance with a prediction reported several years ago by Velarde and Schechter [3, see also 4]. However, the limit of vanishingly small Lewis number cannot be separated from the possibilities allowed to the Soret number. This can be seen as follows. The dimensionless wavenumber is $k = n\pi/f$ where n denotes the number of rolls, then when f goes to infinity, equation (4.11) becomes

$$Ra = (720r_{D}/S)[1 + 59k^{2}/420 + k^{4}/175 + k^{6}/6000]$$
$$\times \{1 + [(6 - 5S)r_{D}/50S + 1/10]k^{2}\}^{-1}. \quad (4.12)$$

Moreover, if we restrict consideration to low enough values of the wavenumber we have the following approximate expression

$$Ra = (720r_{D}/S)\{1 + [17/42 - (6 - 5S)r_{D}/5S]$$

$$\times k^{2}/10 + [(1 + (6 - 5S)r_{D}/5S)$$

$$\times [(6 - 5S)r_{D}/5S - 17/42] + 4/7]$$

$$\times k^{4}/100 + \theta(k^{6})\}.$$
(4.13)

Thus when S is much larger than r_D the asymptotic value of the Rayleigh number follows the law

$$RaS/r_D = 720[1 + 17k^2/420].$$

In this case the critical wavenumber is clearly vanishing, $k^c = 0$, and the corresponding critical Rayleigh number is $Ra^c = 720r_D/S$ in accordance with the known results for infinitely extended horizontal layers [3, 4]. When, however, S is of order r_D the coefficient of k^2 in equation (4.13) can vanish and we ought to proceed to the k^4 term. Then k = 0 although an extremum becomes a local maximum in the neutral stability curve and the minimum is shifted to the new critical wavenumber

$$k^{c} \approx \{(35/4) [(6-5S)r_{D}/5S-17/42]\}^{1/2}$$
 (4.14)

The expansion (4.13) is no longer useful for the description of the onset of instability and we must go back to equation (4.12). Figures 2 and 3 depict the results found with equation (4.12).

Figure 2 describes the role of Lewis number, $r_{\rm D}$. For completeness we have plotted the results found here together with the approximate description given by Velarde and Schechter [3], $Ra^{\rm c}S = 720r_{\rm D}$. Figure 3 illustrates the variation of the critical wavenumber with the Soret effect, S, at given Lewis number, $r_{\rm D}$. For instance, at $r_{\rm D} = 0.01$ the critical wavenumber is vanishingly small provided S is about 0.029 or larger. For lower values of S, $S < r_{\rm D}$, however, the wavenumber shifts to a branch that terminates at $k^{\rm c} = 3.22$ when the problem is reduced to the standard Bénard problem (S = 0).

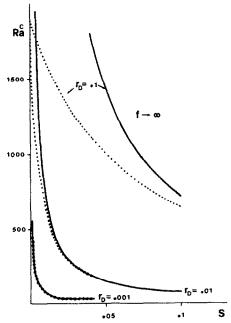


FIG. 2. Neutral stability curves at arbitrarily large values of the aspect ratio (f going to infinity). Dotted lines correspond to equation (4.12) and the solid lines to the approximation of Velarde and Schechter [3] $Ra^{c}S = 720r_{D}$. The dotted lines always cut the vertical axis.

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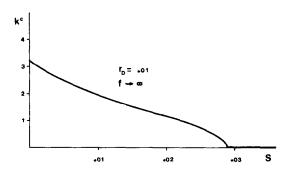


Fig. 3. Critical wavenumber as a function of the Soret separation when the aspect ratio, f, goes to infinity and $r_{\rm D}=0.01$.

(3) The results reported by Crespo and Velarde [1] with a single trial function not only describe quite satisfactorily the experimental data reported by Abernathey and Rosenberger [2] for asymmetric modes of convection but, moreover, with the corrections introduced in this Note they provide a useful description of Bénard convection in cylinders of arbitrary aspect ratio with binary mixtures. For the asymmetric mode of convection these results have been reobtained by an ad hoc heuristic argument by Velarde and Garcia-Ybarra [5]. The latter authors obtain their results by means of an extension of Landau's theory of convective instability to binary mixtures (for an introduction see Velarde and Normand [6] or the review paper by Normand et al. [7]).

(4) Finally, we are pleased to announce that our corrected results have been reobtained in the excellent piece of computation with a large number of trial functions recently

developed by Henry [8]. Thus the latter work constitutes to date the most complete set of quantitative theoretical predictions for both steady and oscillatory two-component Bénard convection in cylinders.

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REFERENCES

- E. Crespo and M. G. Velarde, Two-component Bénard Convection in cylinders, Int. J. Heat Mass Transfer, 25, 1451-1456 (1982).
- J. R. Abernathey and F. Rosenberger, Soret diffusion and convective stability in a closed vertical cylinder, *Phys. Fluids* 24, 377-381 (1981).
- M. G. Velarde and R. S. Schechter, Thermal diffusion and convective stability: an analysis of the convected fluxes, *Phys. Fluids* 15, 1707-1714 (1972).
- R. S. Schechter, M. G. Velarde and J. K. Platten, The twocomponent Bénard problem, Adv. Chem. Phys. 26, 256-301 (1974).
- M. G. Velarde and P. L. García-Ybarra, A model describing Soret diffusion and convective instability in a closed vertical column, J. non-equilib. Thermodyn. 7, 253-258 (1982).
- M. G. Velarde and Ch. Normand, Convection, Sci. Amer. 243, 92–108 (1980).
- Ch. Normand, Y. Pomeau and M. G. Velarde, Convective instability: a physicist's approach, Rev. mod. Phys. 49, 581– 624 (1977).
- D. Henry, Thesis, Université Claude Bernard, Lyon, 1983 (unpublished).